Probability (1)

Probability

Random experiment

Is a trial that leads to an unpredictable outcome.

Example

- Tossing a coin

- Throwing a die

- Have the winning lottery number

Probability

Sample Space (universe)

Set of all possible outcomes (eventualities) of the random experiment.
It is denoted by Ω (read omega)

Example 1

- Tossing a coin

The results are: head (H) or tail (T)

$\Omega = \{H;T\}$

Example 2

- Throwing a die

On the die there are 6 numbers (from 1 to 6), so the number of results is 6.

$\Omega = \{1;2;3;4;5;6\}$

Probability

Event

Is a set of outcomes of the sample space Ω.
It is a part of the sample space Ω.
An event can be represented as a phrase or as a set.

Example

➢ *Throwing a die*

Ω = {1;2;3;4;5;6}

Here are some examples on events:

A: "obtaining an even number"
So A = {2;4;6}

B: "obtaining a multiple of 3
So B = {3;6}

C: "obtaining a number less than 4"
So C = {1;2;3}

Probability

Particular events

- ***Impossible event*** ϕ
  Event that never occur.
- ***Certain event*** Ω
  Event that always occur.
- ***Elementary event***
  Event that contains a single outcome.

Example

➢ ***Throwing a die***

$\Omega = \{1;2;3;4;5;6\}$

A: "obtaining a number greater than or equal to 7"

$A = \phi$

B: "obtaining a number less than or equal to 7"

$B = \{1;2;3;4;5;6\}$

C: "obtaining a multiple of 5"

$C = \{5\}$

Probability

Operations on events

Opposite event (complement)

Let A be an event.
Opposite event of A is the set of outcomes that A does not occur.

It is denoted by : $\overline{A}$

REMARK:

$Card(\Omega) = Card(A) + Card(\overline{A})$

Example

- **Throwing a die**

$\Omega = \{1;2;3;4;5;6\}$

A : "the obtained number is even"

$A = \{2;4;6\}$

$\overline{A}$: "the obtained number is odd"

$\overline{A} = \{1;3;5\}$

Probability

Operations on events

The event (A and B)

Let A and B be 2 events.
The event (A and B) is the set of outcomes that A and B occur simultaneously.

It is denoted by : $A \cap B$

Example

- **Throwing a die**

$\Omega = \{1;2;3;4;5;6\}$

A : "the obtained number is even" $A = \{2;4;6\}$

B : "the obtained number is a multiple of 3" $B = \{3;6\}$

$A \cap B$: "the obtained number is an even number and multiple of 3" $A \cap B = \{6\}$

Probability

Operations on events

→ The event (A and B)

REMARK:

If the two events A and B cannot occur simultaneously, then $A \cap B = \phi$

A and B are said:

"incompatible events"

or

"mutually exclusive events"

$A \cap \overline{A} = \phi$ so A and $\overline{A}$ are incompatible events.

Example

➢ *Throwing a die*

$\Omega = \{1;2;3;4;5;6\}$

A : "the obtained number is even" $A = \{2;4;6\}$

B : "the obtained number is a multiple of 5" $B = \{5\}$

$A \cap B = \phi$

Probability

Operations on events

The event (A or B)

Let A and B be 2 events.
The event (A or B) is the set of outcomes of either A or B or both.

It is denoted by : **A∪B**

REMARK:

$A \cup \overline{A} = \Omega$

Card(A∪B)=card(A)+card(B)-card(A∩B)

Example

➢ *Throwing a die*

Ω = {1;2;3;4;5;6}

A : "the obtained number is even" A = {2;4;6}

B : "the obtained number is a multiple of 3" B = {3;6}

A∪B: "the obtained number is an even number or multiple of 3" A∪B = {2;3;4;6}

Probability

Application # 1

A coin is tossed and its outcomes are observed.

a) Is it possible that head appears?

yes

b) Is it certain that tail appears?

No

c) Is it possible that head and tail appear?

No

d) Is it possible that head or tail appear?

yes

e) Is it impossible that neither head or tail appears?

No

Probability

Application # 2

In a class there are 5 boys and 3 girls. 

The teacher chooses one student randomly from the class.

Given the two events:

G: “the chosen student is a girl”.

T: “the chosen student play Tennis”.

Express in terms of G and T the following events:

- “The student is a boy”

$\bar{G}$

Probability

Application # 2

In a class there are 5 boys and 3 girls. 

The teacher chooses one student randomly from the class.

Given the two events:

G: “the chosen student is a girl”.

T: “the chosen student play Tennis”.

Express in terms of G and T the following events:

- The student doesn’t play tennis”

$\bar{T}$

Probability

Application # 2

In a class there are 5 boys and 3 girls. 

The teacher chooses one student randomly from the class.

Given the two events:

G: “the chosen student is a girl”.

T: “the chosen student play Tennis”.

Express in terms of G and T the following events:

- The student is a girl that plays tennis”

G and T

$G \cap T$

Probability

Application # 2

In a class there are 5 boys and 3 girls. 

The teacher chooses one student randomly from the class.

Given the two events:

G: “the chosen student is a girl”.

T: “the chosen student play Tennis”.

Express in terms of G and T the following events:

- The student is a boy who plays basketball”

$\bar{G}$ and $\bar{T}$

$\bar{G} \cap \bar{T}$

Probability

Application # 2

In a class there are 5 boys and 3 girls. 

The teacher chooses one student randomly from the class.

Given the two events:

G: “the chosen student is a girl”.

T: “the chosen student play Tennis”.

Express in terms of G and T the following events:

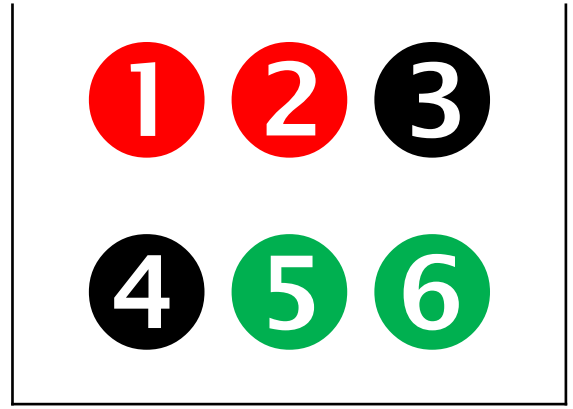
- The student is a girl or plays basketball”

G or $\bar{T}$

$G \cup \bar{T}$

Probability

Application # 3



An urn contains 6 balls: 2 red, 2 black, 2 green.
The red balls are numbered 1, 2.
The black balls are numbered 3, 4.
The green balls are numbered 5, 6.
Two tokens are drawn successively without replacement from the urn.
List all the outcomes of the following events:
A: "the two balls have the same color".

with order

A = {R1R2; R2R1 ; B3B4; B4B3 ; G5G6 ; G6G5}

Probability

Application # 3

An urn contains 6 balls: 2 red, 2 black, 2 green.
The red balls are numbered 1, 2.
The black balls are numbered 3, 4.
The green balls are numbered 5, 6.
Two tokens are drawn successively without replacement from the urn.
List all the outcomes of the following events:
B: "the two balls have different colors".

RB – BR
RG – GR
BG – GB

B = {R1B3; B3R1; R1B4; B4R1; R1G5; G5R1; R1G6; G6R1;
R2B3; B3R2; R2B4; B4R2; R2G5; G5R2; R2G6; G6R2;
B3G5; G5B3; G3B6; B6G3; B4G5; G5B4; B4G6; G6B4}

Probability

Application # 3

An urn contains 6 balls: 2 red, 2 black, 2 green.
The red balls are numbered 1, 2.
The black balls are numbered 3, 4.
The green balls are numbered 5, 6.
Two tokens are drawn successively without replacement from the urn.
List all the outcomes of the following events: BB – GG
C: " the 2 balls are either black or green" GB – BG

C = {B3B4;B4B3;G5G6;G6G5;B3G5;G5B3;B3G6;G6B3;B4G5;G5B4;B4G6;G6B4}

Probability

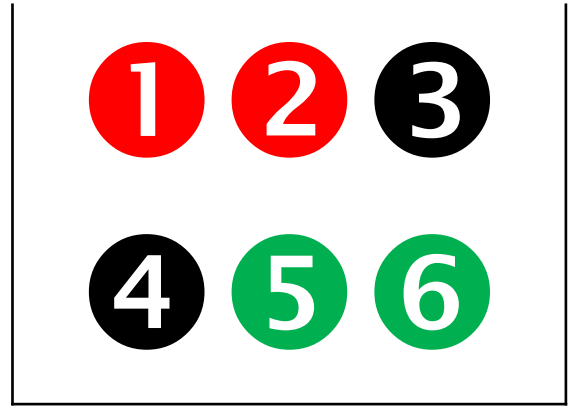
Application # 3

An urn contains 6 balls: 2 red, 2 black, 2 green.
The red balls are numbered 1, 2.
The black balls are numbered 3, 4.
The green balls are numbered 5, 6.
Two tokens are drawn successively without replacement from the urn.
List all the outcomes of the following events:
D: “the two balls have same color and their numbers are less than three”

D = {R1R2; R2R1}

Probability

Application # 3

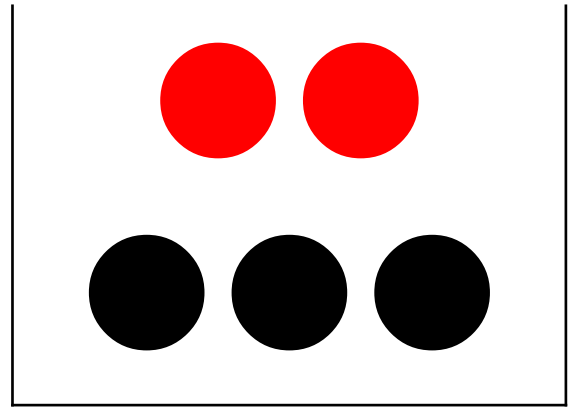


An urn contains 6 balls: 2 red, 2 black, 2 green.
The red balls are numbered 1, 2.
The black balls are numbered 3, 4.
The green balls are numbered 5, 6.
Two tokens are drawn successively without replacement from the urn.
List all the outcomes of the following events:
E: “the two balls have different colors and their numbers are greater than or equal to 4”

E = {B4G5;G5B4;B4G6;G6B4}

Probability

Application # 4



An urn contains 2 red balls and 3 black balls.
Two balls are drawn from the urn one after the other and without replacement.
Answer with true or false.

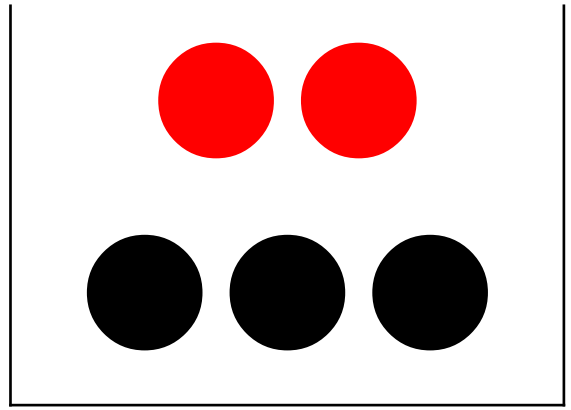
1. The opposite event of the event: “all the balls are red” is “all the balls are black”.

False.

“at least one ball is black”

Probability

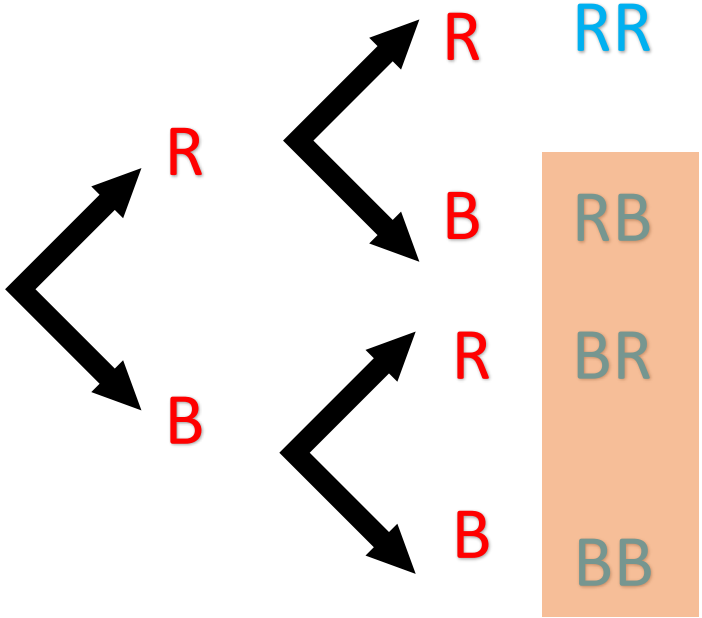
Application # 4



An urn contains 2 red balls and 3 black balls.
Two balls are drawn from the urn one after the other and without replacement.
Answer with true or false.

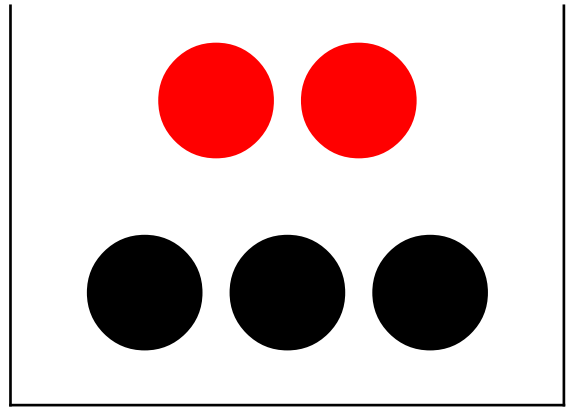
1. The opposite event of the event: "all the balls are red" is "all the balls are black".

False.
"at least one ball is black"



Probability

Application # 4



An urn contains 2 red balls and 3 black balls.
Two balls are drawn from the urn one after the other and without replacement.
Answer with true or false.

2. The event "exactly one ball is red" has only one outcome.

False.

Two outcomes: RB and BR